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Localization and entanglement of two interacting electrons in a double quantum dot

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Abstract

We study the coherent control of two interacting electrons in a coupled-quantum-dot system with external electric fields. The localization and entanglement of the two electrons are demonstrated through analytic and numerical calculations of the populations of the three two-particle states. We show that the maximally entangled Bell state can be prepared and maintained with a pulse of an oscillatory electric field. Although the effective Coulomb repulsion between the two electrons is very strong, dynamical localization can fully build up in the system parameter manifold which corresponds to the exact crossing of the quasienergies developed from the unperturbed nearly degenerate levels.

1. Introduction

Coherent control of quantum systems has been attracting considerable attention in recent years. A basic ingredient of quantum control is field-induced localization of a single electron in a double trap [1]. The initial efforts were devoted to acquiring conditions for maintaining existing localization with an oscillatory electric field, and for creating and maintaining localization with a semi-infinite oscillatory field [2, 3]. It was found that in investigations, localization can be approximated by a two-state model consisting of the lowest symmetric and antisymmetric states of the double-trap potential [4, 5]. In this case, perfect localization can be achieved with a strong time-periodic electric field that causes the Floquet quasienergies to be degenerate. Later, localization in superlattice systems [6], in dissipative environments [7], in molecular systems [8], induced by ultrashort laser pulses, and in trapped Bose–Einstein condensates [9], by means of oscillatory magnetic fields, was studied.

When two or more interacting particles are present, apart from the highly nontrivial problem of whether the strong many-body interaction can be overcome for the particles to create and preserve localization, the possibility of entanglement of the many-body

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wave functions arises. Entanglement is an essential ingredient in any scheme of quantum information processing such as quantum information cryptography and quantum computation, and therefore it is a problem of great current interest to find or design systems where entanglement can be manipulated [10]. Most of the theoretical and experimental activity until now has been associated with atomic and quantum-optic systems. Two-particle [11, 12], three-particle [13], and four-particle [14] entanglement have been successfully demonstrated experimentally for trapped ions, Rydberg atoms, and cavity QED. However, a further increase of the number of entangled particles in these systems is expected to be a severe experimental challenge. Recently, solid-state realizations of the entanglement have received increasing attention due to the fact that semiconductor nanostructures such as quantum dots (QDs) and double quantum dots (DQDs), with well-defined atom-like and molecule-like properties, have been fabricated and studied by many groups [15, 16]. Kane [17] has proposed a scheme which encodes information onto the nuclear spins of donor atoms in doped silicon electronic devices where externally applied electric fields are used to perform logical operations on individual spins. Loss and DiVincenzo [18] have presented a scheme based on spin exchange interaction effects. More recently, Imamoglu *et al* [19] have considered a quantum computer model based on both electron spins and cavity QED which is capable of realizing controlled interactions between two distant quantum dots. Quiroga and Johnson [20] have suggested that the resonant transfer interaction between spatially separated excitons in quantum dots can be exploited to produce maximally entangled Bell states.

In the present work we study the coherent control of the quantum system consisting of two interacting electrons in a coupled quantum dot. With the initial state chosen to be in the spin-singlet space, the dynamics is reduced to be confined to a three-dimensional Hilbert space, in which the three basis vectors are equivalent to the eigenstates of the $\hat{z}$-component, $\hat{J}_z$, of the $J = 1$ angular momentum operator. We show that the maximally entangled Bell state can be prepared and maintained with a pulse of an oscillatory electric field. Also we find that although the Coulomb repulsion between the two electrons is very strong, dynamical localization can fully build up in the system parameter manifold which corresponds to the exact crossing of the quasienergies developed from the unperturbed nearly degenerate levels.

Section 2 begins with a description of our model system. Section 3 examines the dynamics of the initial ground state in the presence of a constant electric field. Section 4 investigates the creation and preservation of the entanglement between the two interacting electrons. Section 5 studies dynamical localization in the presence of an oscillatory field. Section 6 makes some concluding remarks.

2. The model

For simplicity, we suppose there is just one energy level on each quantum dot. The Hamiltonian defining the system reads

$$\hat{H}(t) = \frac{V(t)}{2}\sum_{\sigma}(\hat{c}^{+}_{1\sigma}\hat{c}_{1\sigma} - \hat{c}^{+}_{2\sigma}\hat{c}_{2\sigma}) + w\sum_{\sigma}(\hat{c}^{+}_{1\sigma}\hat{c}_{2\sigma} + \hat{c}^{+}_{2\sigma}\hat{c}_{1\sigma}) + U_1(\hat{n}_{1\uparrow}\hat{n}_{1\downarrow} + \hat{n}_{2\uparrow}\hat{n}_{2\downarrow}) + U_2\sum_{\sigma_1,\sigma_2}\hat{n}_{1\sigma_1}\hat{n}_{2\sigma_2}. \quad (1)$$

Here, $\hat{c}^{+}_{1\sigma}$ ($\hat{c}^{+}_{2\sigma}$) creates an electron of spin σ in the left (right) dot. If the external electric field is applied only to the dots, it will cause a proportionate shift in the energy levels, $\epsilon_{1,2}(t) = \pm V(t)/2$. w describes the coupling between the dots. U_1 and U_2 denote the intradot and interdot Coulomb interactions, respectively. Since there are two electrons in the system,

we can write the Hamiltonian (1) in the space spanned by the two-particle basis vectors $|\bar{1}\,\bar{1}\rangle$, $|11\rangle$, $|1\bar{1}\rangle$, $|\bar{1}1\rangle$, $|20\rangle$, and $|02\rangle$, where $|mn\rangle$ denotes the state of m electrons in the left dot and n electrons in the right dot, and the values 1 and $\bar{1}$ indicate up spin and down spin, respectively. Therefore the Hamiltonian is described by a 6×6 matrix. If we replace the basis vectors $|\bar{1}1\rangle$ and $|1\bar{1}\rangle$ by $(|\bar{1}1\rangle - |1\bar{1}\rangle)/\sqrt{2}$ and $(|\bar{1}1\rangle + |1\bar{1}\rangle)/\sqrt{2}$, then the Hamiltonian can be written as

$$\hat{H}(t) = \begin{pmatrix} U_2\hat{I}_{3\times 3} & 0 \\ 0 & \hat{H}_1(t) \end{pmatrix} = U_2\hat{I}_{3\times 3} \oplus \hat{H}_1(t) \tag{2}$$

where $\hat{I}_{3\times 3}$ is a 3×3 unit matrix and $\hat{H}_1(t)$ is presented as

$$\hat{H}_1(t) = \begin{pmatrix} U_1 + V(t) & \sqrt{2}w & 0 \\ \sqrt{2}w & U_2 & \sqrt{2}w \\ 0 & \sqrt{2}w & U_1 - V(t) \end{pmatrix}. \tag{3}$$

Obviously the two-particle basis vectors $|\bar{1}\,\bar{1}\rangle$, $|11\rangle$, and $(|\bar{1}1\rangle - |1\bar{1}\rangle)/\sqrt{2}$ are the eigenvectors of the Hamiltonian (2) and constitute the spin-triplet subspace in which the electron number on each quantum dot is invariably one and has no response to the presence of the driving electric field. Hence we will focus our attention on the reduced spin-singlet Hamiltonian $\hat{H}_1(t)$. Note that $\hat{H}_1(t)$ can be conveniently rewritten in terms of the angular momentum operators in the $J = 1$ subspace:

$$\hat{H}_1(t) = U_2 + V(t)\hat{J}_z + \kappa\hat{J}_z^2 + 2w\hat{J}_x \tag{4}$$

where $\kappa = U_1 - U_2$, and $\hat{J}_i$ $(i = x, y, z)$ are $J = 1$ angular momentum operators defined as

$$\hat{J}_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\hat{J}_y = \frac{\mathrm{i}}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

and

$$\hat{J}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Therefore, the localized two-particle state $|20\rangle$ is equivalent to the eigenstate $|j_z = 1\rangle$ of $\hat{J}_z$ and $|02\rangle$ to the state $|j_z = -1\rangle$, whereas the delocalized state $(|\bar{1}1\rangle + |1\bar{1}\rangle)/\sqrt{2}$ is identical to the state $|j_z = 0\rangle$. In the following we will denote $|20\rangle$ as $|\mathrm{LL}\rangle$, $|02\rangle$ as $|\mathrm{RR}\rangle$, and $(|\bar{1}1\rangle + |1\bar{1}\rangle)/\sqrt{2}$ as $|\mathrm{LR}\rangle$.

The first term in equation (4) denotes a constant energy shift, and will be neglected in the following discussions. The evolution of any initial state $|\Psi(0)\rangle$ under the action of $\hat{H}_1$ in equation (4) can be expressed as

$$|\Psi(t)\rangle = C_1(t)|\mathrm{LL}\rangle + C_2(t)|\mathrm{LR}\rangle + C_3(t)|\mathrm{RR}\rangle$$

where the coefficients $C_k(t)$ are determined by the time-dependent Schrödinger equation

$$\mathrm{i}\begin{pmatrix} \dot{C}_1 \\ \dot{C}_2 \\ \dot{C}_3 \end{pmatrix} = \begin{pmatrix} \kappa + V(t) & \sqrt{2}w & 0 \\ \sqrt{2}w & 0 & \sqrt{2}w \\ 0 & \sqrt{2}w & \kappa - V(t) \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} \tag{5}$$

and the chosen initial condition $|\Psi(0)\rangle$. In the absence of external electric fields, the eigenenergies and eigenstates (not normalized) of the Hamiltonian $\hat{H}_1$ can be easily solved as follows:

$$\begin{aligned}
\left|\varphi_1^{(\mathrm{S})}\right\rangle &= |\mathrm{LL}\rangle - \frac{b}{\sqrt{2}w}|\mathrm{LR}\rangle + |\mathrm{RR}\rangle \qquad & E_1 &= a \\
\left|\varphi_2^{(\mathrm{A})}\right\rangle &= -|\mathrm{LL}\rangle + |\mathrm{RR}\rangle & E_2 &= \kappa \\
\left|\varphi_3^{(\mathrm{S})}\right\rangle &= |\mathrm{LL}\rangle - \frac{a}{\sqrt{2}w}|\mathrm{LR}\rangle + |\mathrm{RR}\rangle & E_3 &= b
\end{aligned} \tag{6}$$

where we have defined

$$a = (\kappa - \sqrt{\kappa^2 + 16w^2})/2 \qquad b = (\sqrt{\kappa^2 + 16w^2} + \kappa)/2.$$

The superscript S (A) on the left-hand sides of equation (6) denotes symmetry (antisymmetry) under the spatial reflection operation. We can see from equation (6) that due to strong Coulomb repulsion, the symmetric ground state is dominated by the delocalized state $|\mathrm{LR}\rangle$, whereas the other two eigenstates are nearly degenerate and dominated by the two localized states $|\mathrm{LL}\rangle$ and $|\mathrm{RR}\rangle$. Note that although $|\varphi_2^{(\mathrm{A})}\rangle$ and $|\varphi_3^{(\mathrm{S})}\rangle$ look like a doublet in a single-electron double-trap system which consists of a pair of symmetric and antisymmetric single-particle states, there are fundamental differences for the present two-particle system. In fact, the superposition of the two localized states $|\mathrm{LL}\rangle$ and $|\mathrm{RR}\rangle$ implies that the spatial wave functions of the two electrons have been entangled, in the usual sense that they are not factorized into single-particle states. To describe the degree of entanglement, we define the maximally entangled Bell state $|\Psi_{Bell}\rangle = (|\mathrm{RR}\rangle + \mathrm{e}^{\mathrm{i}\phi}|\mathrm{LL}\rangle)/\sqrt{2}$ with arbitrary phase angle ϕ. Therefore the probability ρ_{Bell} of finding the maximally entangled Bell state in a coupled quantum dot is given by

$$\rho_{Bell} = \frac{1}{2}\left|C_3(t) + \mathrm{e}^{\mathrm{i}\phi}C_1(t)\right|^2. \tag{7}$$

3. Localization preparation from the ground state

We start the search for localization with the simplest case, that of a constant electric field $V(t) = V_0$. Before $t = 0$ the system is in the delocalized ground state $|\varphi_1^{(\mathrm{S})}\rangle$, and at $t = 0$ the field V_0 is switched on suddenly. The eigenenergies and eigenstates associated with the time-independent Hamiltonian (4) can be solved analytically for general values of V_0. For brevity we do not give the explicit expressions here. Instead, we illustrate in figure 1 the spectrum features by plotting the eigenenergies as a function of V_0 ($w = 0.02\kappa$). It is shown in figure 1 that on increasing the value of V_0 adiabatically, the energies E_1 and E_2 approach each other. In particular, when the value of the electric field satisfies

$$V_0 = \kappa \tag{8}$$

an avoided crossing occurs in the energy spectrum. To elucidate the effect of the avoided crossing displayed in figure 1 on the quantum mechanical behaviour of the system we examine the dynamics of the initial ground state. Figures 2(a)–2(e) show the time evolutions of the probabilities $P_{\mathrm{LR}}(t) = |C_2(t)|^2$ of finding the two electrons in the different dots (solid lines), $P_{\mathrm{LL}}(t) = |C_1(t)|^2$ of finding the two electrons in the left dot (dashed lines), and $P_{\mathrm{RR}}(t) = |C_3(t)|^2$ of finding the electrons in the right dots (dotted lines). The values of V_0/κ in these five figures are respectively 0.8, 0.9, 1.0, 1.1, and 1.2. It is revealed in figures 2(a)–2(e) that on changing the value of the constant field V_0 towards the avoided crossing, the oscillation amplitudes of $P_{\mathrm{LR}}(t)$ and $P_{\mathrm{RR}}(t)$ increase more and more—until, at the avoided crossing point

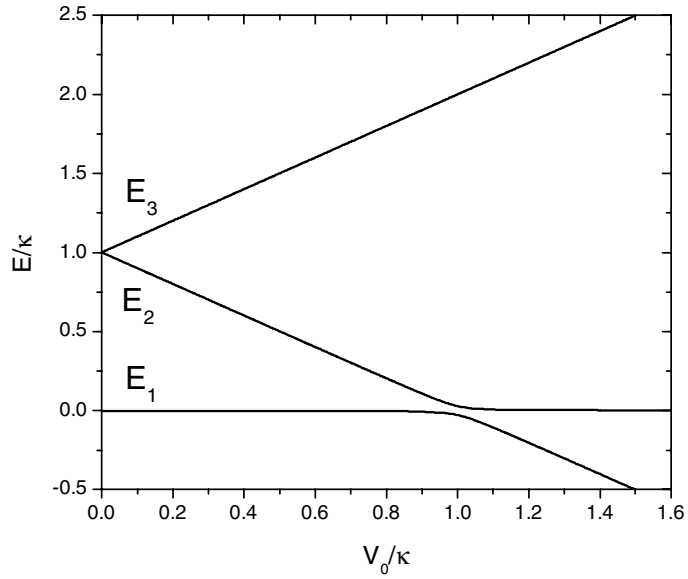

Figure 1. The energy spectrum of the driven two-electron system as a function of the strength of a constant electric field with $w = 0.02\kappa$.

where $V_0/\kappa = 1$, both $P_{\mathrm{LR}}(t)$ and $P_{\mathrm{RR}}(t)$ oscillate between 0 and 1 with a definite period, meaning that a complete resonance (Rabi oscillation) takes place between the delocalized state $|\mathrm{LR}\rangle$ and localized state $|\mathrm{RR}\rangle$. After the value of V_0 exceeds κ, the oscillations decrease little by little. This represents a localization–delocalization transition process. The most important feature revealed in figure 2 is that when the system parameters are chosen near the avoided crossing, the dynamics can be approximated by an effective two-state system consisting of $|\mathrm{LR}\rangle$ and $|\mathrm{RR}\rangle$. In this case, because the population of the localized state $|\mathrm{LL}\rangle$ in the left dot remains very small during time evolution, we can neglect its contribution and describe the dynamics by the reduced Schrödinger equation

$$\mathrm{i}\begin{pmatrix}\dot{C}_2\\ \dot{C}_3\end{pmatrix} = \begin{pmatrix}0 & \sqrt{2}w\\ \sqrt{2}w & 0\end{pmatrix}\begin{pmatrix}C_2(t)\\ C_3(t)\end{pmatrix}. \tag{9}$$

Thus with the initial state $|\Psi(0)\rangle = |\varphi_1^{(\mathrm{S})}\rangle \simeq -|\mathrm{LR}\rangle$, we have the time evolution of the system as follows:

$$\begin{aligned} C_2(t) &= -\cos(\sqrt{2}wt)\\ C_3(t) &= \exp(-\mathrm{i}\pi/2)\sin(\sqrt{2}wt). \end{aligned} \tag{10}$$

Clearly our two-state approximation, equation (10), describes the system's evolution very well when compared with the exact numerical result shown in figure 2(c), implying complete Rabi oscillation between the localized state $|\mathrm{RR}\rangle$ and delocalized state $|\mathrm{LR}\rangle$ with oscillation period $T = \pi/\sqrt{2}w$.

Once the two electrons are localized, they can be forced to stay localized permanently by switching the field to another nonzero value (figure 3, top panel). We show this effect in figure 3 (bottom panel). This trivial way of maintaining localization in a two-electron double-dot system originates from the fact that, due to the presence of an electric field, the energy mismatch among the three two-particle states prohibits tunnelling from the localized state $|\mathrm{RR}\rangle$ to the other states.

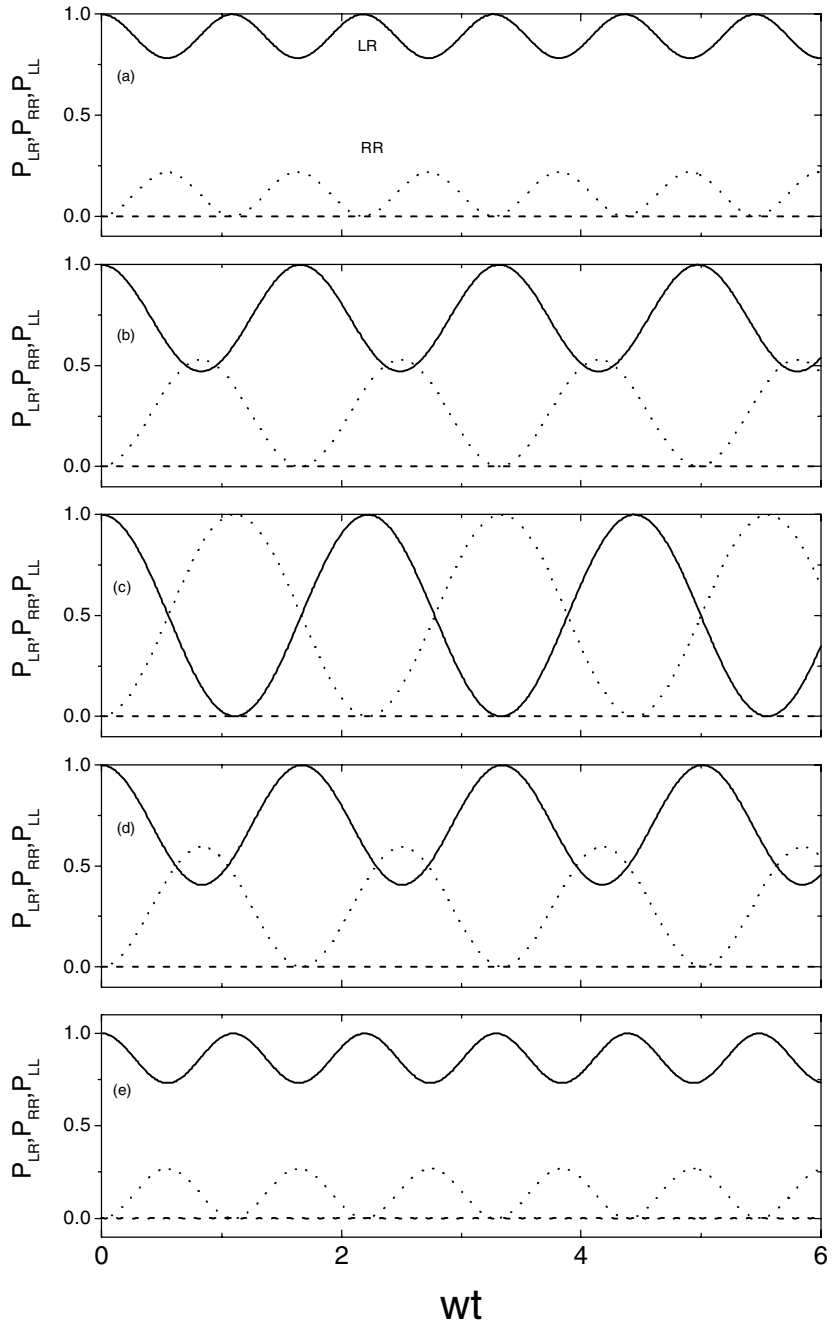

Figure 2. The time evolution of the probabilities P_{LR} (solid lines), P_{LL} (dashed lines), and P_{RR} (dotted lines) for the following values of the strength of a constant electric field: (a) $V_0 = 0.8\kappa$; (b) $V_0 = 0.9\kappa$; (c) $V_0 = 1.0\kappa$; (d) $V_0 = 1.1\kappa$; (e) $V_0 = 1.2\kappa$. For these figures, $w = 0.02\kappa$.

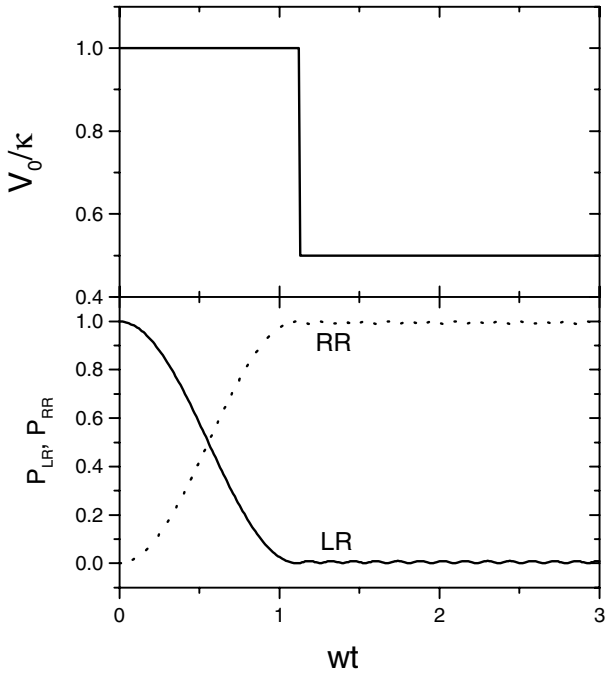

Figure 3. Top: the electric field that is imposed on the double-quantum-dot system. Bottom: the time evolution of the probabilities P_{LR} (solid line) and P_{RR} (dotted line) under the influence of the electric field shown in the top panel. The other system parameters are the same as those used for figure 2(c).

4. Entanglement of two interacting electrons

If the constant electric field is turned off (figure 4(a)) at a time when the two electrons are fully localized in the right dot, as shown in figure 2(c), the strong Coulomb repulsion will induce the resonance between the two localized states $|\mathrm{RR}\rangle$ and $|\mathrm{LL}\rangle$ during subsequent evolution, whereas the delocalized state $|\mathrm{LR}\rangle$ is inhibited from being occupied. This dark property of the delocalized state is shown in figure 4(b). It is revealed in figure 4(b) that the value of P_{LR} is almost zero during the time evolution, suggesting that the two electrons are never separated into different dots. While cycling from one dot to the other, the two electrons are correlated and entangled, and very likely to be found in the same dot.

The entanglement between the two electrons illustrated in figure 4(b) can be well described by a two-state approximation of equation (5). Because the population of the delocalized state $|\mathrm{LR}\rangle$ remains very small after time $t_0 = T/2$ as shown in figure 4(b), we can approximate $C_2(t)$ $(t > t_0)$ in equation (5) to first order of w/κ:

$$C_2(t) = \frac{-\sqrt{2}w}{\kappa} \exp(-\mathrm{i}\kappa t)[C_1(t) + C_3(t)]. \tag{11}$$

By introducing $C_2(t)$ from equation (11) in the Schrödinger equation we reduce the system to an effective two-level system. The reduced equation has the form

$$\mathrm{i}\begin{pmatrix} \dot{C}_1 \\ \dot{C}_3 \end{pmatrix} = \begin{pmatrix} \kappa & -2w^2/\kappa \\ -2w^2/\kappa & \kappa \end{pmatrix} \begin{pmatrix} C_1(t) \\ C_3(t) \end{pmatrix}. \tag{12}$$

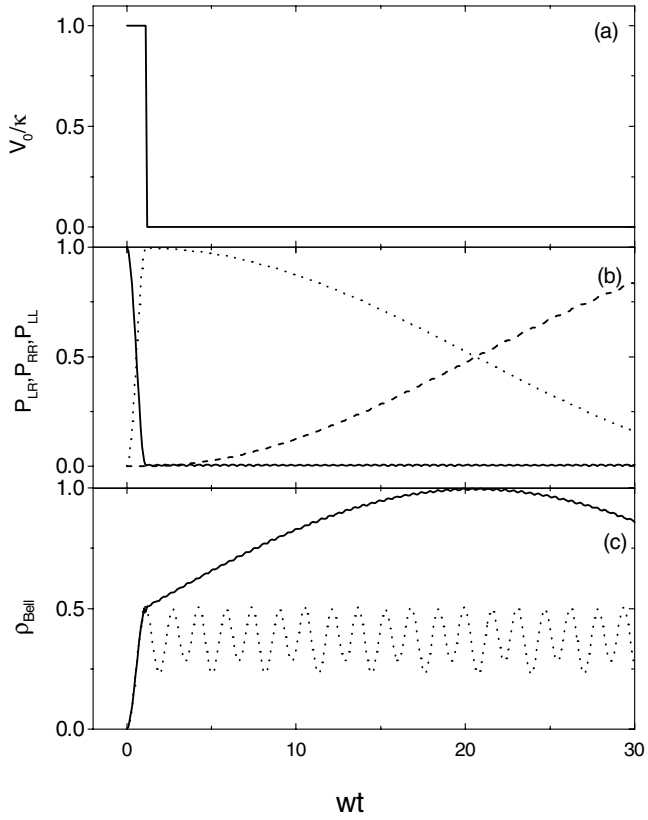

Figure 4. (a) The electric field that is imposed on the double-quantum-dot system. (b) The time evolution of the probabilities P_{LR} (solid line), P_{LL} (dashed line), and P_{RR} (dotted line) under the influence of the electric field shown in (a). (c) The time evolution of the probability ρ_{Bell} of finding the maximally entangled Bell state ($\phi = -\pi/2$) in the presence of the Coulomb interaction (solid line) and in the absence of the Coulomb interaction (dotted line). For these figures, $w = 0.02\kappa$.

Thus with the initial state $|\Psi(t_0)\rangle = -\mathrm{i}|\mathrm{RR}\rangle$ (equation (10)), we have the following time evolution of the system:

$$\begin{aligned} C_1(t) &= \exp(-\mathrm{i}\kappa t)\sin(2w^2 t/\kappa) \\ C_3(t) &= -\mathrm{i}\exp(-\mathrm{i}\kappa t)\cos(2w^2 t/\kappa). \end{aligned} \tag{13}$$

Substituting equation (13) into equation (7) we have the probability of finding the Bell state $(|\mathrm{RR}\rangle + \mathrm{e}^{\mathrm{i}\phi}|\mathrm{LL}\rangle)/\sqrt{2}$ at time t:

$$\rho_{Bell}(t) = \frac{1}{2}[1 + \sin(\omega_r t)\cos(\phi + \pi/2)] \tag{14}$$

where $\omega_r = 4w^2/\kappa$. In particular we can see from equations (13), (14) that the system's quantum state at time

$$\tau = \pi\kappa/8w^2 + t_0 \tag{15}$$

corresponds to a $\phi = -\pi/2$ maximally entangled Bell state $(|\mathrm{RR}\rangle - \mathrm{i}|\mathrm{LL}\rangle)/\sqrt{2}$.

We present in figure 4(c) the probability of finding the maximally entangled Bell state ($\phi = -\pi/2$) as a function of time. The system parameters are the same as those used

in figure 4(b). Clearly our two-state approximation (equation (14)) describes the system's evolution very well as compared with the exact numerical solution shown in figure 4(c), implying that the system's quantum state at time τ corresponds to a maximally entangled Bell state ($\phi = -\pi/2$). However, the degree of entanglement degrades after time τ, a consequence of the fact that the state at time τ is not an eigenstate of the field-free Hamiltonian. So a single pulse of a constant electric field cannot preserve the entanglement in our system. Note that no maximally entangled Bell-state generation is possible if the effective Coulomb interaction κ, along with the electric field, is turned off, as shown in figure 4(c) (dotted line). This implies an essential role of the nonlinear Coulomb interaction in forming the entanglement between the electrons.

We turn now to discussion of the entanglement in the presence of a sinusoidal field of the form $V(t) = V_1 \cos \omega t$. We present in figures 5(a)–5(e) populations of the three two-particle states, where the values of ω/κ in these five figures are respectively 0.9, 0.95, 1.0, 1.05, and 1.1. It is revealed in figure 5 that, contrary to the case for constant field shown in figure 2, where only P_{RR} increased at the expense of P_{LR}, the reduction of P_{LR} results in an increase of both P_{RR} and P_{LL} with the same in-phase oscillations. Also we can see in figure 5 that on increasing the value of the driving frequency ω towards the Coulomb interaction κ, the oscillation amplitudes of P_{RR} and P_{LL} increase little by little—until when the value of ω satisfies the one-photon resonance condition

$$\omega = \kappa \tag{16}$$

the two amplitudes reach their maximal values 0.5, which implies maximal entanglement between the two electrons at the time when $P_{\mathrm{RR}} = P_{\mathrm{LL}} = 0.5$.

Once the two electrons are in the maximally entangled Bell state, they can be kept maximally entangled by suddenly turning off the oscillatory electric field (figure 6(a)). We show this effect in figures 6(b), 6(c) where figure 6(b) plots the time evolution of the occupations of three two-particle states and figure 6(c) the probability of finding the maximally entangled Bell state with $\phi = \pi$. It is revealed in figure 6(b) that a pulse of an oscillatory electric field induces the two electrons to stay at the same dot, while each of them occupies either of the dots with the same probability. It is shown in figure 6(c) that the two electrons remain maximally entangled during the time evolution after the oscillatory field is turned off. This is different from the case shown in figure 4(b) where the degree of entanglement varies with time. The control of the maximally entangled quantum states in solid-state systems is of great interest. Loss and Di Vincenzo studied entanglement in double quantum dots involving the spin degree of freedom [18]. Here we have identified a complementary method that creates and preserves entanglement between the spatial wave functions of two electrons in a coupled quantum dot.

5. Dynamical localization of two interacting electrons

In section 3 we showed that a trivial way of maintaining localization is to suddenly shift the constant field to another value once the electrons are in the localized state $|\mathrm{RR}\rangle$. In this section we study the possibility of maintaining the localization with the oscillatory electric field $V(t) = V_1 \cos \omega t$. Because the localized state $|\mathrm{RR}\rangle$ may always be produced from the ground state by turning on a resonant constant field of duration $\pi/\sqrt{2}w$, we therefore suppose in the following discussions that the system starts with the localized state $|\mathrm{RR}\rangle$.

In the presence of a time-dependent electric field, the evolution of the system cannot be solved in a closed form because $[H_1(t_1), H_1(t_2)] \neq 0$. Within the Floquet formalism, we numerically integrate the equation of motion for the time evolution operator:

$$\mathrm{i}\frac{\partial}{\partial t}\hat{U}(t,0) = \hat{H}_1(t)\hat{U}(t,0) \tag{17}$$

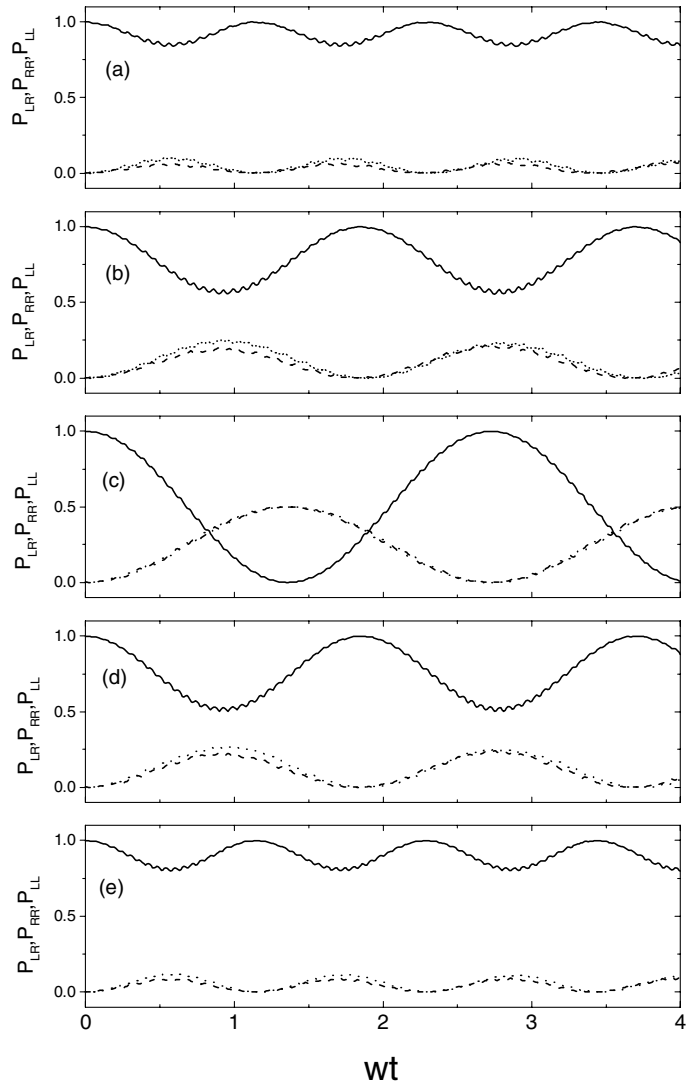

Figure 5. The time evolution of the probabilities P_{LR} (solid lines), P_{LL} (dashed lines), and P_{RR} (dotted lines) for the following values of the frequency of an oscillatory electric field: (a) $\omega = 0.9\kappa$; (b) $\omega = 0.95\kappa$; (c) $\omega = 1.0\kappa$; (d) $\omega = 1.05\kappa$; (e) $\omega = 1.1\kappa$. The other system parameters are $V = 2.0\kappa$ and $w = 0.02\kappa$.

and diagonalize $\hat{U}(2\pi/\omega, 0)$ to obtain the quasienergies $\{\varepsilon_{\alpha,l}\}$ and Floquet states $\{|u_{\alpha,l}(0)\rangle\}$ at time $t = 0$. Here the quasienergies $\varepsilon_{\alpha,l}$ are confined to the first Brillouin zone and, at $V_1 = 0$, connected to $E_\alpha + l\omega$. The index l counts 'how many photons' have to be subtracted from the unperturbed energy level E_α in order to arrive in the first Brillouin zone. The Floquet state $|u_{\alpha,l}(t)\rangle$ can be obtained from the eigenvalue equation

$$\left(H_1(t) - \mathrm{i}\,\frac{\partial}{\partial t}\right)|u_{\alpha,l}(t)\rangle = \varepsilon_{\alpha,l}|u_{\alpha,l}(t)\rangle \tag{18}$$

where $\alpha = 1, 2, 3$. Note that the Hamiltonian (4) remains invariant under the combined spatial reflection and time translation $t \to t + \pi/\omega$. An immediate consequence of this dynamical

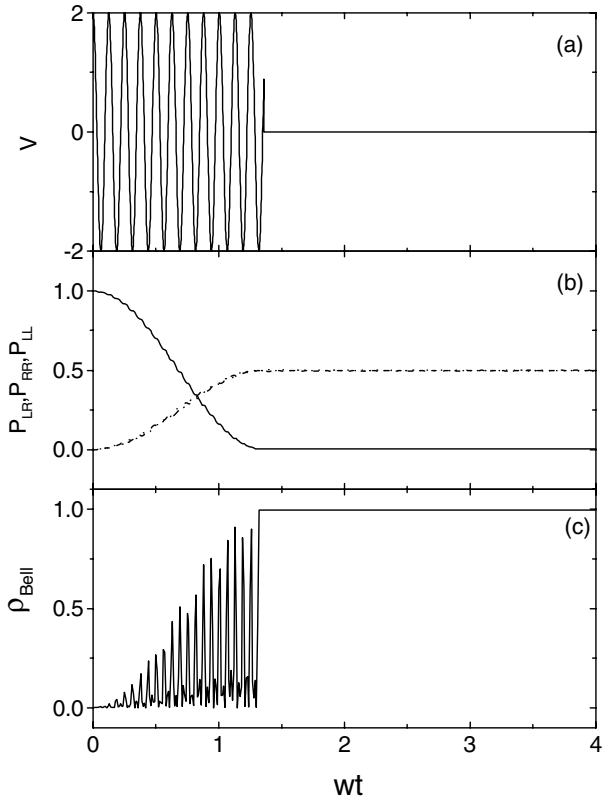

Figure 6. (a) The electric field that is imposed on the double-quantum-dot system. (b) The time evolution of the probabilities P_{LR} (solid line), P_{LL} (dashed line), and P_{RR} (dotted line) under the influence of the electric field shown in (a). (c) The time evolution of the probability ρ_{Bell} of finding the maximally entangled Bell state ($\phi = \pi$). The other system parameters are the same as those used for figure 5(c).

symmetry is that each Floquet state is either odd or even [21, 22]. When the driving amplitude is switched off adiabatically, $V_1 \to 0$, the Floquet states are connected with the stationary eigenstates in equation (6) as follows [1]:

$$|u_{\alpha,l}(t)\rangle \to \left|u^0_{\alpha,l}(t)\right\rangle = \varphi_\alpha \exp(\mathrm{i}l\omega t). \tag{19}$$

Thus we can easily determine the dynamical parity of the Floquet state $|u_{\alpha,l}(t)\rangle$.

We present in figures 7(a), 7(b) the quasienergies versus the amplitude V_0, where the values of effective Coulomb interaction κ are respectively chosen to be 2.3ω and 2.84ω. In figure 7(a) we see that the quasienergies $\varepsilon_{2,-2}$ and $\varepsilon_{3,-2}$ with different parity form an exact crossing at $V_0 = 1.244\,48\,\omega$. When $V_0 \to 0$, all three quasienergies arrive at the unperturbed cases for $\varepsilon^0_{1,0} = E_1$, $\varepsilon^0_{2,-2} = E_2 - 2\omega$, $\varepsilon^0_{3,-2} = E_3 - 2\omega$. For $(2l+1)\omega \to E_3 - E_1$, $\varepsilon_{1,0}$ and $\varepsilon_{3,-(2l+1)}$, having different parity, are also allowed to cross, as shown in figure 7(b) where the value of the index is $l = 1$. In exactly the same way, when $2l\omega \to E_2 - E_1$, two quasienergies $\varepsilon_{1,0}$ and $\varepsilon_{2,-2l}$ can develop into a crossing for a special value of the amplitude V_0 (not shown here). In addition, the characteristic double-cone structure in figure 7(b) reveals that the quasienergies $\varepsilon_{1,0}$ and $\varepsilon_{2,-3}$ form an avoided crossing with the centre at $V_0 = 1.053\,83\,\omega$. This avoided crossing originates as a result of interaction between the Floquet states $|u_{1,0}\rangle$

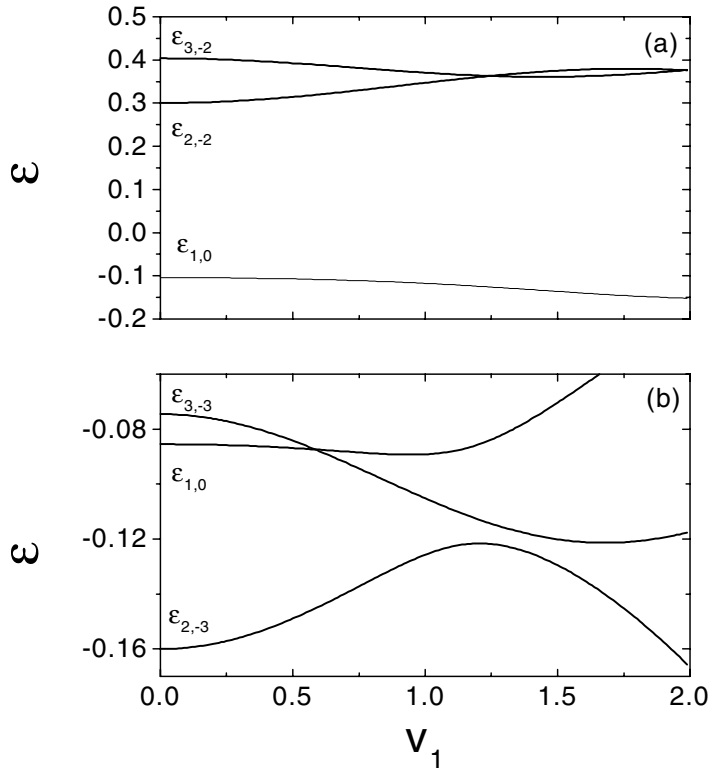

Figure 7. The Floquet spectrum of the driven two-electron system as a function of the strength of the oscillatory electric field for the following values of the effective Coulomb interaction: (a) $\kappa = 2.3\omega$; (b) $\kappa = 2.84\omega$. For these figures, $w = 0.25\omega$.

and $|u_{2,-3}\rangle$, both having the same parity. Similarly, when $2l\omega \to E_3 - E_1$, two quasienergies $\varepsilon_{1,0}$ and $\varepsilon_{3,-2l}$ are allowed to develop into the avoided crossing.

To elucidate the effect of the exact crossing on the quantum mechanical behaviour of the system, we present in figure 8(a) the time evolution of $P_{\mathrm{RR}}(t)$ at integer multiples of the driving period with the system parameters corresponding to the exact crossing of $\varepsilon_{2,-2}$ and $\varepsilon_{3,-2}$ shown in figure 7(a). For comparison, we also present in figure 8(b) the time evolution of $P_{\mathrm{RR}}(t)$ for the value of the amplitude $V_0 = 1.1\omega$. In figure 8(a) we can see that during the development over time the probability $P_{\mathrm{RR}}(nT)$ remains near 1 as if the two electrons were frozen in the left dot. Moreover, time-resolved evolution over a few periods of the driving field (not depicted) shows that the two electrons stay localized also at times $t \neq nT$. Thus at the exact crossing of $\varepsilon_{2,-2}$ and $\varepsilon_{3,-2}$ the dynamical localization builds up, although the strong on-site Coulomb repulsion between the two electrons prevents the system from behaving in this way. Note that the value κ used in figure 7(a) corresponds to the unperturbed energies $E_1 = -0.1034\omega$, $E_2 = 2.3\omega$, and $E_3 = 2.4034\omega$. Because E_1 is much lower than E_2 and E_3, the unperturbed eigenstates $\varphi_2^{(\mathrm{A})}$ and $\varphi_3^{(\mathrm{S})}$ which both have very small components of the delocalized two-particle state $|\mathrm{LR}\rangle$ become comparable to a doublet in the symmetrical double-dot system. So it is expected that at the crossing of the quasienergies $\varepsilon_{2,-2}$ and $\varepsilon_{3,-2}$ the initial localized state can be approximated by a superposition of degenerate Floquet states $|u_{2,-2}(0)\rangle$ and $|u_{3,-2}(0)\rangle$, which remains localized in perpetuity. This is like the case of a single-electron,

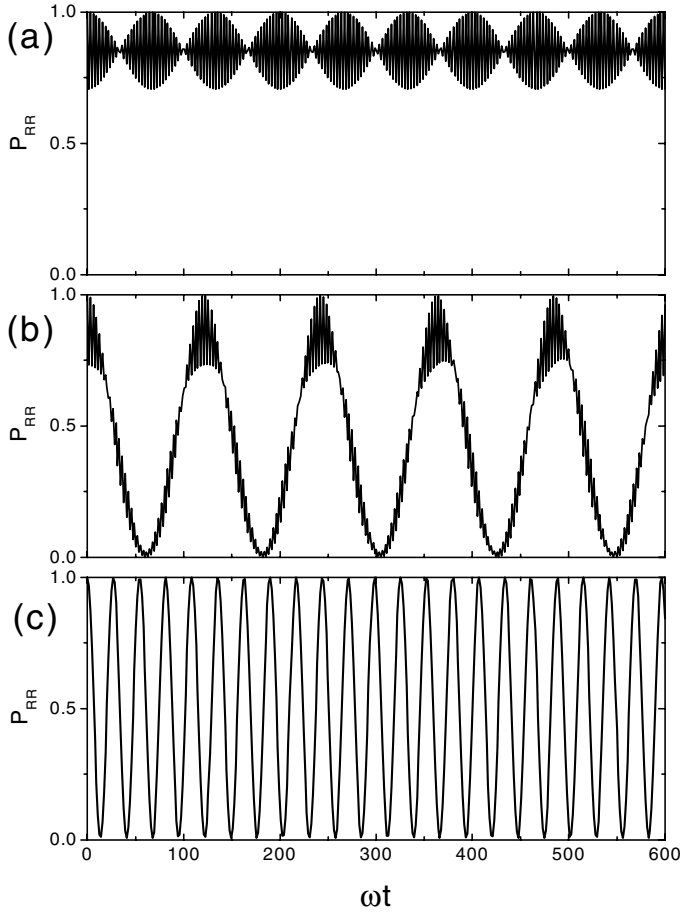

Figure 8. The time evolution of the probability P_{RR} at integer multiples of the driving period for three different kinds of system parameter values: (a) $\kappa = 2.3$ and $V_1 = 1.244\,48$, corresponding to the exact level crossing shown in figure 7(a); (b) $\kappa = 2.3$ and $V_1 = 1.1$, for comparison with the case shown in (a); (c) $\kappa = 2.84$ and $V_1 = 0.5834$, corresponding to the exact level crossing shown in figure 7(b).

two-level system consisting of the lowest symmetric and antisymmetric states of the double-trap potential, where perfect localization can be achieved at the exact crossing between the two Floquet quasienergies. It is found numerically that even if the Coulomb interaction κ is very strong, the dynamical localization can still occur as long as the quasienergies $\varepsilon_{2,m}$ and $\varepsilon_{3,m}$ cross each other. Moreover, the value of the system parameter $2V_0/\omega$ corresponding to the first crossing is about 2.4, which is the root of the zero-order Bessel function, suggesting that in this situation the dynamical localization can be approximated by the driven two-level model. If the system parameters deviate from the level crossing, then the dynamical localization ceases to exist and $P_{\mathrm{RR}}(t)$ oscillates between 0 and 1 in the development over time, as shown in figure 8(b).

We turn to studying the dynamics of the system at the crossing of the quasienergies $\varepsilon_{1,0}$ and $\varepsilon_{3,-3}$ in figure 7(b), using the same initial-state condition. The result is shown in figure 8(c). Contrary to that shown in figure 8(a), the result shown in figure 8(c) indicates that at the

level crossing of $\varepsilon_{1,0}$ and $\varepsilon_{3,-3}$, dynamical localization does not happen and $P_{\mathrm{RR}}(t)$ oscillates between 0 and 1. Note that the level crossing of $\varepsilon_{1,0}$ and $\varepsilon_{3,-3}$ induces strong participation of the Floquet state $|u_{1,0}(t)\rangle$ during the time evolution of the system, and the largest component in $|u_{1,0}(t)\rangle$ is the delocalized two-particle state $|\mathrm{LR}\rangle$. Therefore the strong mixture of the Floquet state $|u_{1,0}(t)\rangle$ in the evolution of the system will lead to complete destruction of the dynamical localization, as shown in figure 8(c).

In the above discussions we have ignored higher-lying single-particle states; this requires that the frequency of the external field is much lower than the single-particle level spacing. In the presence of decoherence due to environmental dissipation, a long dephasing time would be required. A detailed analysis of the effect of a decohering environment will be given elsewhere.

6. Conclusions

In conclusion, we have studied the localization and entanglement of two interacting electrons in a double-quantum-dot system. We have shown that:

(i) The presence of a constant electric field satisfying the resonance condition $V_0 = \kappa$ induces complete Rabi oscillation between the delocalized state $|\mathrm{LR}\rangle$ and the localized state $|\mathrm{RR}\rangle$. Thus, starting from the delocalized ground state, we can prepare a fully localized state. The localization can be maintained by switching the field to another nonzero value.

(ii) The two electrons oscillate between the delocalized state and two localized states in the presence of a resonant oscillatory field. With the oscillatory field turned off at a time when the probabilities of finding the electrons in the left and right dot are identically 0.5, the two electrons remain maximally entangled in the subsequent time evolution. Thus a selective pulse of an oscillatory field can be used to implement maximally entangled Bell states in a two-electron two-dot system.

(iii) Although the Coulomb repulsion is very strong, the two initially localized electrons can stay localized during the time evolution. We expect the present results to be useful in exploiting the coherent control of electrons in quantum dot systems.

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